Math 301 — Limit theorems for sequences

Summary. In first semester calculus, we learn useful algebraic rules for working with limits. You might remember them phrased as statements like "the limit of a sum is the sum of the limits." Proving these algebraic statements is good practice with common analysis techniques.

Problem 1. Fill in the blanks in the proof of the algebraic limit theorem for sums (part 2). Use the following list to fill in the blanks. Some items might not get used and some might get used more than once.

a. ϵ

b. $\epsilon/2$

c. there exists N_a

d. there exists N_b

e. $|a_n - a| + |b_n - b|$

f. $n > \max\{N_a, N_b\}$

g. $n > N_a$

h. $n > N_b$

i. for all n

Proof. Let $\epsilon > 0$. Since (a_n) converges to a, `BLANK1` so that

$$|a_n - a| < \texttt{BLANK2}$$

when `BLANK3`. Similarly, since (b_n) converges to b, `BLANK4` so that

$$|a_n - a| < \texttt{BLANK5}$$

when `BLANK6`. Therefore, when `BLANK7`, we have that

$$\begin{aligned} |(a_n + b_n) - (a + b)| &= |(a_n - a) + (b_n - b)| \\ &\le \texttt{BLANK8} \\ &< \texttt{BLANK9}. \end{aligned}$$

□

Problem 2. Give a proof of the scalar algebraic limit theorem (part 1).