

# Math 339SP, Fall 2025 — Homework 5

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Due October 9 at 5:00 pm

**Instructions.** This problem set contains problems mostly from Week 5 of class. The problem numbers refer to our textbook, *Introduction to Stochastic Processes with R* by Robert P. Dobrow.

**Problem 1.** Please do the following textbook problems: 3.11b, 3.15, 3.18, 3.19, 3.31, 3.33.

**Problem 2.** Consider a Markov chain with transition matrix  $P$  and two recurrent communication classes  $C_1$  and  $C_2$ , with 3 states and 4 states respectively. Suppose that the transition probabilities among  $C_1$  are described by the regular  $3 \times 3$  matrix  $P_1$ , and, similarly, transitions among  $C_2$  are described by the regular  $4 \times 4$  matrix  $P_2$ . Let  $a, b, c$  be the expected first return times for the states in  $C_1$  and let  $d, e, f, g$  be the expected first return times for the states in  $C_2$ . Find as many entries of  $\lim_{n \rightarrow \infty} P^n$  as possible.

*Remark 1.* In Exercise 3.15, note that a chess board has 64 squares. Also, note that the knight is allowed to make moves in an ‘L’ shape (2 units horizontally and 1 unit vertically or 2 units vertically and 1 unit horizontally). Try thinking of the Markov chain in this problem as a random walk on a graph, where the vertices are the 64 squares and there is an edge connecting two vertices if the knight is allowed to move from one of those squares to the other.

*Remark 2.* In preparation for Exercises 3.18 and 3.19, make sure to read about First Step Analysis and work through Example 3.17 in Section 3.4.

*Remark 3.* In Exercise 3.19, exploiting the symmetries in the graph means using the fact that symmetry in the graph implies  $e_b = e_f$  and  $e_c = e_e$  where  $e_x = E[T_d \mid X_0 = x]$ .

*Remark 4.* In preparation for Exercise 3.31, make sure to read Example 3.21 in Section 3.6.