Math 339SP, Fall 2025 — Homework 8

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Due November 6 at 5:00 pm

Instructions. This problem set contains problems mostly from Week 9 of class. The problem numbers refer to our textbook, *Introduction to Stochastic Processes with R* by Robert P. Dobrow.

Problem 1. Please do the following textbook problems: 6.7, 6.8, 6.14, 6.15, 6.16, 6.24, 6.27, 6.29

Remark 1. In Exercises 6.8d and 6.14, you might find it helpful to condition on the inter-arrival time between two successive arrivals. Remember that if T is a continuous random variable with density $f(t)$ that is non-zero on $(0,\infty)$ and A is an event of interest, then by the continuous law of total probability,

$$P(A) = \int_0^\infty P(A \mid T = t) f(t)\, dt.$$

This is what is meant by conditioning on the value of T.

Remark 2. In Exericse 6.14, the goal is to show for each $k \geq 0$, $P(X = k) = (1-p)^k p$ for some value of $p \in (0,1)$ that you must determine. When using the remark above, you'll probably find it helpful to note that for every real number $\lambda > 0$ and every integer $n \geq 1$ it is the case that

$$\int_0^\infty \frac{\lambda^n t^{n-1} e^{-\lambda t}}{(n-1)!}\, dt = 1.$$

This identity is a consequence of the fact that the integrand is the density of the Gamma distribution with parameters n and λ. It can be proved using integration by parts and induction on n.