

Math 339SP, Fall 2025 — Homework 9

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Due November 13 at 5:00 pm

Instructions. This problem set contains problems mostly from Week 10 of class. The problem numbers refer to our textbook, *Introduction to Stochastic Processes with R* by Robert P. Dobrow.

Problem 1. Please do the following textbook problems: 7.1, 7.2

Problem 2. During lunch hour, customers arrive at a fast-food restaurant at a rate of 120 customers per hour. The restaurant has one line, with three workers taking orders at independent service stations. Each worker takes an exponentially distributed amount of time to service a customer with an average service time of 1 minute. Let X_t denote the number of customers in the restaurant, in line and being served, at time t in minutes.

1. Give the state space of this Markov chain.
2. Draw the transition rate diagram and label transition rates.
3. Give the hold time distribution for each state.
4. Give the embedded chain transition matrix \tilde{P} .

Problem 3. For the fast-food restaurant of the previous problem, assume that customers turn away from the store if all three service stations are busy. Let Y_t denote the number of service stations busy at time t . Please answer the same questions as the previous problem.

Problem 4. Consider the Markov chain with transition rate diagram given in Figure 7.12 on page 315.

1. Give the hold time distribution for each state.
2. Give the embedded chain transition matrix \tilde{P} .

Problem 5. In class we learned about the $M/M/1$ queue model, where arrivals occur according to a Poisson process with rate λ , service times are independent and exponentially distributed with parameter μ , there is 1 server, and X_t denotes the number of customers in line or being serviced at time t . Draw the transition rate diagram and label transition rates for the $M/M/c$ queue where arrivals and service times are modeled at the same rates, but there are $c \in \mathbb{N}$ servers that give service to customers in parallel and independently.