Math 339SP — Introduction

Problem 1. For each of the following examples, identify a stochastic process $\{X_t : t \in I\}$. Then, determine the state space $\mathcal{S}$ and the index set I.

a. We're interested in modeling day to day weather; specifically, we're interested in rain conditions. Suppose that the chance of rain tomorrow depends only on today's conditions and not on any past conditions. If it rains today, then it will rain tomorrow with probability 0.6. If it does not rain today, then it will rain tomorrow with probability 0.2.

b. A gambler starts with \$5 and decides to play the following game. In each round, a coin is tossed. If the coin lands on heads, the gambler wins \$1; otherwise the gambler loses \$1. The gambler stops playing when they run out of money or when they have \$10 (that is, \$5 more than what they started with).

c. A small grocery store has one employee whose job is to work the cashier. When a customer arrives at the cashier line, if no one is in line, they are served. Any other customers that arrive at the line while the employee is busy must wait to be served. However, if the line is too long (ie. there are 10 or more people), they leave. What if there's no limit to the length of the line?

Problem 2. Let's focus on the weather example above. Part c below might feel a little tricky but we'll review probability ideas together all semester and we'll learn new ideas related to matrix algebra that will make these and more complex questions more routine.

a. Draw a transition state diagram and write the transition matrix for this example.

b. Suppose today is Day 0 and it's raining. Given this, what is the probability that it's raining on Day 1? There is no computation to do here, but I want you express what we're looking for using random variable and conditional probability notation.

c. Suppose today is Day 0 and there is a 30% chance it's raining. What is the (unconditional) probability that it's raining on Day 1? Before giving a numerical answer, express what we're looking for and what we're given using random variable notation. Try using a tree diagram, or more preferably the Law of Total Probability, to find an answer.

Problem 3. Let's focus on the gambler example above.

a. Draw a transition state diagram and write the transition matrix for this example.

b. If the gambler starts with \$5, what is the probability they have \$5 after two rounds of the game?

c. Suppose the gambler starts with a random amount of money, either \$5, 6, or 7 with equal probability. What is the probability they have \$5 after one round of the game?