

Math 339SP, Spring 2022 — Ergodicity

Class on March 1

The following questions try to get us to understand the ideas of aperiodicity, ergodicity, and the limit theorem for ergodic Markov chains.

Problem 1. Consider the random walk on the graph below. Why is this Markov chain ergodic? What is its limiting distribution?

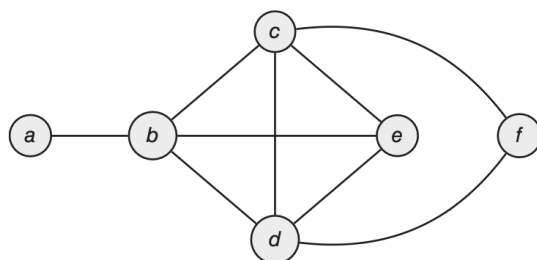


Figure 2.1 Graph on six vertices.

Problem 2. A Markov chain, with transition matrix P has k states. Before each step a biased coin with heads probability p is tossed. If the coin lands heads, the chain stays at its current state. If the coin lands tails, the chain moves to any of the k states uniformly at random. Why is this Markov chain ergodic? What is its limiting distribution?

Problem 3. Consider the Markov chain with transition matrix below. Describe how it evolves over time. Why is this Markov chain ergodic?

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \cdots & k-2 & k-1 & k \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ k-2 \\ k-1 \\ k \end{matrix} & \begin{pmatrix} 1/k & 1/k & 1/k & \cdots & 1/k & 1/k & 1/k \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{pmatrix} \end{matrix} .$$