Math 339SP — Arrival times

We've seen that a Poisson process counts the number of arrivals in a given time period. Last time we asked questions about how many arrivals happened. Now we also ask questions about the times when arrivals happen or the time gaps between arrivals.

Problem 1. Customers arrive at a store starting at 6:00 am according to a Poisson process with a mean rate of 8 customers per hour. Each of the following events can be expressed in two ways: (a) in terms of N_t, (b) in terms of arrival or inter-arrival times. For each event, use whichever is easier and then use R to compute the probability of the event. If you have time, you can try both methods and check that your answers agree.

a. No customer comes in the first hour of the store opening.

b. There is at least 30 minutes between the arrivals of the first and second customers.

c. There is at least 2 hours between the arrivals of the third and fifth customers.

d. The 50th customer comes in after 1:00 pm.

e. In the first 5 hours, at least 40 customers come in.

Problem 2. At a certain intersection, red, green, and orange cars go by according to three independent Poisson processes. On average there is one red car every 10 minutes, one green car every 15 minutes, and one orange car every 20 minutes.

a. What are the parameters $\lambda_R, \lambda_G, \lambda_O$ of the Poisson processes, using minutes as time units?

b. What is the probability that you wait at least 15 minutes between seeing your fourth and fifth orange car pass?

c. What is the probability that the third green car you see goes by between minutes 25 and 30 of you watching?

d. You've already waited 40 minutes for a red car without seeing one. Given this, what is the probability you'll need to wait at least an additional 15 minutes before seeing a red car? What is the expected additional time you'll wait for a red car?

e. Bonus questions that preview some ideas to come:

   1. What is the probability of waiting more than 10 minutes for a car, of any color, to arrive? How long will you wait on average before a car, of any color, arrives?[1]
   2. What is the probability that the first car to arrive is green?[2]

[1] Hint: there are two ways to approach this question: (1) define a new Poisson process which models the arrivals of any car of red, green, or orange color, or (2) find the distribution of the minimum of the first arrival time of the red, green, and orange car processes; that is find the distribution of $M = \min\{X_R, X_G, X_O\}$.

[2] Hint: can you use the random variable M defined in the previous hint?