Math 339SP, Spring 2022 — Transition rates

Class on April 7

For each of the following examples of continuous time Markov chains (CTMC), answer the following questions:

1. Draw a transition diagram showing the possible transitions that can occur between states. Label the edges with transition rates.
2. What is the hold-time parameter for each state i?
3. What is the transition matrix $\widetilde{P}$ for the embedded chain?

Example 1. Consider two independent machines that are maintained by a single person. Each machine functions for an exponentially distributed amount of time with parameter λ before breaking down. The repair time for either machine is exponentially distributed with parameter μ. Let X_t denote the number of broken machines at time t.

Example 2. Suppose in the previous example that there are 2 maintenance people. The time it takes either of them to repair a machine is exponentially distributed with parameter μ. Suppose that if only one machine is broken, one of them repairs it and the other is idle. If two machines are broken, then they can work simultaneously, but independently, on each machine. Let X_t denote the number of broken machines at time t.

Example 3. Consider a population where members can give birth to new members but cannot die. Suppose each member acts independently and takes an exponentially distributed amount of time, with mean $1/\lambda$, to produce an offspring. Let X_t be the population size at time t.

Example 4. Like in the last example, consider a population where each member acts independently and takes an exponentially distributed amount of time, with mean $1/\lambda$, to produce an offspring. Further, suppose that the lifespan of each member is exponentially distributed, with mean $1/\mu$. is Let X_t be the population size at time t.