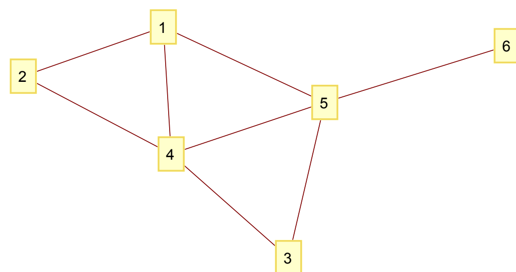
Math 339SP, Spring 2022 — Transition probabilities

Class on January 25



Consider the following Markov chain, called a **random walk on a graph**:

- A random walker starts at a vertex of a given graph.
- At each discrete time unit, the walker moves to a neighbor vertex; each neighbor equally likely.
- Let X_k denote the walker's location after k steps.

Problem 1. Try the following exercises using the graph shown above.

1. Find the transition matrix P.
2. What do the following conditional probabilities mean in words? How many time units elapse? Between which states do you transition? What does the notion of *time-homogeneity* tell us about parts b and c?
   (a) $P(X_2 = 3 \mid X_0 = 1)$
   (b) $P(X_7 = 5 \mid X_4 = 4)$
   (c) $P(X_{50} = 5 \mid X_{40} = 2)$
3. Try computing the probability in part a.

Problem 2. Let's think more generally now. Suppose we're working with a Markov chain whose state space is $\mathcal{S} = \{1, 2, \dots, m\}$. Its transition matrix P is an $m \times m$ matrix and its ij-entry is $P_{ij} = P(X_1 = j \mid X_0 = i)$. This is all given to us. Try using the Conditional Law of Total Probability to write an expression for

$$P(X_2 = j \mid X_0 = i),$$

called the 2-step transition probability. Your expression should be a summation and its terms should be written using entries of the matrix P. Here's a question that might feel tricky: what is the matrix algebraic meaning of the expression you found? If you figure these questions out, try thinking about $P(X_3 = j \mid X_0 = i)$, called the 3-step transition probability.