

Math 339SP, Spring 2022 — Limiting distributions

Class on February 8

Problem 1. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/4 & 3/4 \\ 2/3 & 1/3 \end{bmatrix}.$$

Use R to make a conjecture about whether this Markov chain has a limiting distribution. If you believe it does, tell me what it is, at least approximately.

Problem 2. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Draw the transition state diagram for this Markov chain. Do you think the Markov chain has a limiting distribution? Do you think $\lim_{n \rightarrow \infty} P^n$ exists? What makes this Markov chain qualitatively different from the previous one? Use R to support and think through your ideas.

Problem 3. A *6-cycle* is a graph with 6 vertices that are arranged in a circle so that each vertex has two neighbors. Consider the random walk on a 6-cycle. Its transition matrix is given by

$$P = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 \end{bmatrix}.$$

Do you think the Markov chain has a limiting distribution? Do you think $\lim_{n \rightarrow \infty} P^n$ exists? What makes this Markov chain qualitatively different from the previous two?