# Math 339SP, Spring 2022 — Communication classes

Class on February 15

**Problem 1.** For each of the transition matrices below, draw the transition state diagram for the chain and then find the communication classes. That is, partition the state space into disjoint sets of states that communicate with each other.

$$P_1 = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/2 \end{bmatrix},$$

$$P_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1/3 & 0 & 0 & 2/3 & 0 & 0 \\ 0 & 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 1/7 & 0 & 0 & 6/7 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/3 & 0 & 2/3 \end{bmatrix} \end{array}.$$

**Problem 2.** We will soon learn that for any communication class, states in that class are either all recurrent or all transient. For each communication class you found in the Markov chains above, decide whether its states are transient or recurrent.

**Problem 3.** For transition matrix $P_3$ you should find that the communication classes are $\{1,4\}, \{2,5\}, \{3\}$, and $\{6\}$. Suppose we rearrange the rows and columns so that states in the same communication class are adjacent as below. Fill in the entries in the matrix below. What do you notice about the block structure? Discuss how you think this rearranging of states (what we will learn is called the *canonical decomposition*) and reconfiguring of the matrix $P_3$ (what we will learn is called the *canonical form* of $P_3$) helps you understand $\lim_{n\to\infty} P_3^n$.

$$P_3 = \begin{array}{c} \\ 3 \\ 6 \\ 1 \\ 4 \\ 2 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 3 & 6 & 1 & 4 & 2 & 5 \end{array} \\ \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{array}.$$