Math 339SP, Spring 2022 — Canonical decomposition

Class on February 22

The following questions try to get us to justify and understand the structure of the canonical form of the transition matrix of a finite state Markov chain.

Problem 1. Consider a Markov chain with communication class C. Suppose C contains at least one recurrent state. Why is it the case that any state $j \notin C$ fails to be accessible from all the states within C? What would happen if there was a state $i \in C$ that could transition to j?

Problem 2. Consider a Markov chain with transition matrix P whose state space is finite and consists of two communication classes C_1 and C_2, each with recurrent states. Explain the structure of the canonical form of P using block matrices and the previous problem.

Problem 3. Consider a Markov chain with transition matrix P whose state space is finite and consists of two communication classes C_1 and C_2, one transient and one recurrent, respectively. Explain the structure of the canonical form of P using block matrices. What can be said about $\lim_{n\to\infty} P^n$ if you know that the block matrix P_2 for the transition probabilities among the recurrent states in C_2 is a regular matrix?

Problem 4. Consider a Markov chain with transition matrix P whose state space is finite and consists of three communication classes C_1, C_2, and C_3, one transient and two recurrent. Explain the structure of the canonical form of P using block matrices. What can be said about $\lim_{n\to\infty} P^n$ if the block matrices of the recurrent classes are regular?