

Math 342 —Negative binomial distribution

Problem 1. Suppose applicants for a job are hired with probability $p = 0.15$ independently from person to person, one at a time as they come in.

- a. Find the probability of each of the following events by first expressing the event in terms of a negative binomially distributed random variable.
 1. It took exactly 10 applicants until 3 people were hired.
 2. It took at least 10 applicants until 3 people were hired.
 3. It took exactly 15 applicants until 7 people were hired.
 4. It took at least 15 applicants until 7 people were hired.
- b. Express each of the following events in terms of a binomially distributed random variable.
 1. It took exactly 15 applicants until 7 people were hired.
 2. It took at least 15 applicants until 7 people were hired.
- c. Find the expected value of the number of applicants seen until:
 1. 3 hires are made.
 2. 7 hires are made.

Problem 2. Alice and Bob are playing a game. In each round of the game, one of them wins the round: Alice wins the round with probability $1/3$ and Bob wins with probability $2/3$. Whoever wins 50 rounds first wins the overall game. However, the game has to end prematurely before either of them gets to 50 wins. Alice needed to win 5 more rounds and Bob had to win 10 more rounds to reach 50.

- a. What is the maximum number of rounds they could have played before declaring an overall winner of the game?
- b. Suppose they played out this maximum number of rounds. Find the probability that
 1. Alice would have won 5 more rounds in the maximum number of possible rounds or fewer.
 2. Bob would have won 10 more rounds in the maximum number of possible rounds or fewer.
- c. Who is more likely to have won if the game had not been ended prematurely?

Remark. This is an example of what is commonly called the problem of points. The original formulation of the problem was solved jointly by Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665) in a series of letters