

Math 342 — Exam 1 review

Your exam in class on March 1 will contain about 5 multi-part problems. It will cover material from Homework 0 to Homework 4. In the textbook, this is material spanning sections 1.1 through 4.2, though we've skipped some material in that range which will not be part of the exam. There will be no material related to R or Monte Carlo simulation on the exam. The problems below give you a sampling of some problems like those that will appear on the exam, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes. There are also suggested exercises that have been given as part of each homework, most of which have answers at the back of the textbook. No notes will be allowed on the exam, but there will be some formulas given on the exam sheet. You'll be allowed to use a scientific calculator with no graphing or calculus functionality, but the problems will be written so that a calculator is not required.

Problem 1. Suppose that A , B , and C are events in an experiment, with C and $A \cup B$ mutually exclusive and

$$P(AB^c) = 1/6, \quad P(BA^c) = 1/4, \quad P(AB) = 1/12, \quad P(C) = 5/12$$

Find the probability of each of the following:

- A
- at least one of A or B occurs
- exactly one of the three events occurs
- all three events occur
- at least one of the three events occurs

Problem 2. A symphony orchestra has in its repertoire 30 Haydn pieces, 15 modern pieces, and 9 Beethoven pieces. A program consists of three different pieces from the repertoire. Suppose we choose a program at random. Find the probability that the program has

- two modern pieces
- more than one piece of the same type
- a Hayden piece first, followed by 2 modern pieces

Problem 3. A bag of Scrabble tiles contains two of each of the letters R, A, N, D, O, and M for a total of 12 tiles. Six tiles are picked without replacement and placed left to right on a Scrabble rack. Find the probability that you:

- spell R-A-N-D-O-M from left to right
- pick both R's
- pick no M's

Problem 4. Every Saturday afternoon Carmen plays golf with probability 0.3 or plays squash with probability 0.7. After the golf game, she goes out for a massage with probability 0.55, and after the squash game, she goes out for a massage with probability 0.2.

- Find the probability that she will go out for a massage.
- If she goes out for a massage, what is the probability that she played golf?

Problem 5. There are three coins in a box. One is two-headed, one is fair, and one is biased to come up heads with probability 0.75. A coin is selected at random, flipped, and shows heads. What is the probability that it was the two-headed coin?

Problem 6. Suppose A, B, C are independent events with respective probabilities $1/6$, $1/4$, and $1/2$. Find the probability that

- at least one of the events occurs
- A does not occur, given that both B and C occur
- A and B occur, given that A or B occur

Problem 7. A coin has heads probability $1/3$.

- Find the probability that among 7 tosses of the coin
 - no heads appear
 - exactly 3 heads appear
 - at least 5 heads appear
- Suppose 5 people each make 7 tosses of the coin. Find the probability that at least 3 of them get no heads.

Problem 8. The number of times that a person contracts a cold in a given year is Poisson distributed. The probability they contract no colds in a year is 30%. Find the probability that over the next 2 years, the person contracts exactly 5 colds.

Problem 9. A hotel has 25 single rooms, all occupied, and numbered 1 to 25. Each room, independent of the others, makes on average 3 phone calls per hour.

- Give an appropriate distribution for the number of phone calls made by a single room in a one hour period, including any relevant parameters.
- Give an appropriate distribution for the number of rooms which each made exactly 2 phone calls in a one hour period, including any relevant parameters.
- Find the probability that
 - at least 1 phone call was made by room 5 in a one hour period.
 - at least 2 phone calls were made in total by rooms 5 and 6 in a one hour period.
 - at least 3 rooms each made exactly 2 phone calls on a one hour period.

Problem 10. Consider a random variable X whose probability mass function is given below. Find the following quantities.

$$P(X = x) = \begin{cases} c & x = -3 \\ 2c & x = -2 \\ 2c & x = 0 \\ 3c & x = 1 \\ 4c & x = 2. \end{cases}$$

- c
- $P(X > -2)$
- $P(-2 \leq X < 2)$
- $E[X]$
- $E[X^2]$