

Math 342 —Densities of Transformed Random Variables

Problem 1. Let $X \sim \text{Exp}(\lambda)$ and let $Y = cX$. Find the density f_Y of Y and state its distribution.

Problem 2. Let $X \sim \text{Exp}(1)$ and let $Y = \sqrt{X}$. Find the density f_Y of Y .

Problem 3. The density, and hence distribution, of Y in Problem 2 is probably not familiar to you, but we can check our answer with simulation. The following steps will show you how make a large (say 1 million element) i.i.d. sample of Y random variables, plot the histogram of the sample, and overlay the density curve you found to see if matches the histogram. In an RMarkdown code chunk do the following:

- Use the command `x = rexp(1e6, 1)` to generate 1 million random numbers sampled from the $\text{Exp}(1)$ distribution.
- Use the command `y = sqrt(x)` to take the square root of each element of our sample.
- Use the command `hist(y, prob = T)` to make a histogram of our sample.
- Use the command `curve(..., add = T, col = "red")` to overlay your proposed density. For example, if you believe the density is $f_Y(x) = 5x^2e^x$ then in place of `...` you will put `5*x^2*exp(x)`.

Problem 4. Let

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

be a given density function and let

$$F(x) = \begin{cases} 0 & x \leq 0 \\ x^3 & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}$$

be its corresponding cumulative distribution function.

- State $F^{-1}(x)$ for $0 < x < 1$.
- Let $U \sim \text{Unif}(0, 1)$ and $Y = F^{-1}(U)$. Find the density of Y .
- Use R to simulate 1 million samples of Y . Note that the R command `runif(1e6, 0, 1)` will let you simulate 1 million samples of U .
- Make a histogram of your samples of Y and overlay its density.