

Math 342 —Max and Min of Independent Random Variables

Problem 1. Suppose we pick 4 random numbers in the interval $(0, 1)$ independently. Let M_1 be their minimum and let M_2 be their maximum.

- a. Find the CDFs of M_1 and M_2 .
- b. Find the densities of M_1 and M_2 .
- c. Find the probability that the smallest number is greater than $1/4$.
- d. Find the probability that the biggest number is less than $1/2$.
- e. Find the expected value of the smallest number.
- f. Find the expected value of the biggest number.

Problem 2. Suppose the lifetime for a certain brand of lightbulb is modeled with the exponential distribution with a mean lifetime of 5 years. Suppose further that we buy 10 such bulbs and assume that their lifetimes are independent. Let M be the time until the first one dies.

- a. Find $P(M > 5)$.
- b. Find $E[M]$.