Math 342 — Central Limit Theorem

Problem 1. Let $X_1, X_2, \ldots$ be an i.i.d. sequence of random variables with probability mass function

$$P(X_i = k) = \begin{cases} 0.6 & k = +1 \\ 0.4 & k = -1. \end{cases}$$

Think of each X_i as the outcome of one round of a game where you win or lose \$1 with a slight bias to win \$1 on each round. Use the Central Limit Theorem and the `pnorm` command in R to approximate the probability that after 40 rounds of the game your net winnings are between \$4 and \$6.

Problem 2. Consider a continuous distribution with probability density function

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Suppose you go into R and generate 30 random numbers from this distribution, sampling independently. Use the Central Limit Theorem and the `pnorm` command in R to approximate the probability that the sample average of your 30 random numbers is in the interval $(0.7, 0.8)$.

Problem 3. Suppose you have invited 64 guests to a party and need to determine how much food to buy. You believe that each guest will eat 0, 1, or 2 sandwiches with probability 1/6, 1/2, and 1/3 respectively. Assume that the number of sandwiches each guest eats is independent from other guests.

a. Use the Central Limit Theorem and the `pnorm` command in R to approximate the probability that your guests eat less than 75 sandwiches in total.

b. The 95th percentile of the $N(\mu, \sigma^2)$ distribution is the number $q \in \mathbb{R}$ defined so that if $X \sim N(\mu, \sigma^2)$ then $P(X \leq q) = 0.95$. Within R, you can find the 95th percentile (or other percentiles) using the command `qnorm(0.95, mu, sigma)`. Use this concept to find the fewest number of sandwiches you should buy so that there is at most a 5% chance of having a shortage of sandwiches.